Department of Electronic Engineering
ELE2EMI
Electronic Measurements & Instrumentation

3 Basic Op-Amp Circuit Analysis and Design

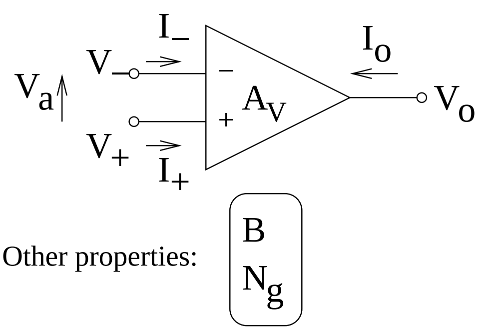
Chapter reference:

- *Carr*, chapter 12.

3.1 Outline

- Differential op-amp symbol and properties
- Ideal op-amp characteristics
- Ideal op-amp circuit analysis
- Real op-amp characteristics
- Offset Currents and Voltages
- Compensating for Offsets
- Basic applications of the op-amp

3.2 Differential op-amp symbol and properties



The differential op-amp has two inputs and one output. Its purpose is to produce a voltage V_o that is a multiple A_V of the differential input voltage $V_a = V_- - V_+$. The voltage gain A_V should be large.

Although they are not always shown in schematics, every op-amp is connected to an upper power supply V_{cc} (which should be positive and greater than any normal output voltage) and a lower power supply V_{EE} (which should be negative and lower than the op-amp's output).

3.3 Ideal Op-Amp Characteristics

- Infinite open loop voltage gain, $A_V = \infty$
- Infinite input impedance, $Z_{in} = \infty$
- Zero output impedance, $Z_{out} = 0$
- Infinite bandwidth, $B = \infty$
- Zero noise generation, $N_g = 0$
- Power supply voltages are infinite: $V_{CC} = +\infty$ and $V_{EE} = -\infty$

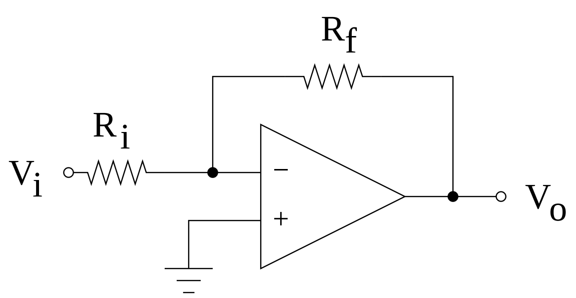
3.4 Ideal Op-Amp Circuit Analysis

What are the consequences of these characteristics?

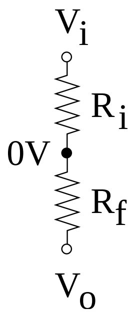
- Since the voltage gain A_V of the ideal operational amplifier (op-amp) is infinite and its output voltage V_o must be finite, therefore its differential input voltage V_i is zero; in other words $V_+ = V_-$.
- Since its input impedance Z_{in} is infinite, its input currents I_+ and I_- are both zero.
- Zero output impedance implies that the output voltage V_o is unaffected by the *load* circuit to which it is delivered.
- Infinite bandwidth means that the op-amp works identically at all frequencies, "from DC to daylight" as the saying goes.
- The ideal op-amp is stable; this requires that it produce no noise internally; indeed, it must not permit any noise to enter it from any source, otherwise its infinite voltage gain would amplify the noise infinitely, producing extreme instability.
- *Ideally* the power supplies have infinite voltage, since any finite output voltage V_o is assumed possible.

3.4.1 Inverting amplifier circuit

A typical inverting amplifier circuit is shown below.



Since V_+ is grounded, $V_- = 0$ also; and since $I_- = 0$, the currents in R_1 and R_f are equal. So the resistors form a voltage divider:



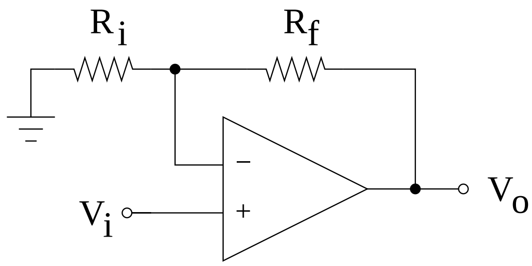
from which we deduce the voltage gain,

$$\frac{V_o}{V_i} = -\frac{R_f}{R_i}$$

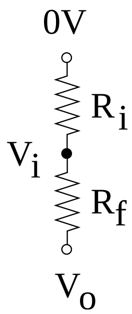
which implies that the circuit does indeed invert the input signal. Note that the gain< 1 if $R_f < R_i$ so we usually choose $R_f > R_i$.

3.4.2 Non-inverting amplifer circuit

A non-inverting amplifier circuit,



is similar, but the positions of the input V_i and ground $0V$ voltages have been reversed, leading to this voltage divider:



and this voltage gain:

$$\frac{V_o}{V_i} = 1 + \frac{R_f}{R_i}$$

which ≥ 1 no matter what the resistances are.

3.5 Real Op-Amp Characteristics

- Finite open loop voltage gain, $A_V < \infty$
- Finite input impedance, $Z_{in} < \infty$
- Nonzero output impedance, $Z_{out} \neq 0$
- Finite bandwidth, $B < \infty$
- Nonzero noise generation, $N_g > 0$
- Finite supply voltages: $0 < V_{CC} < +\infty$ and $-\infty < V_{EE} < 0$

Interestingly, the *gain-bandwidth product* GB (where G is another symbol for A_V) is nearly constant for each specific type of op-amp.

Since in real-life the power supply voltages V_{CC} and V_{EE} are finite, any output voltage V_o had best keep between them, otherwise it will be clipped. (This is called *hard limiting*.)

In fact, as V_o approaches one of the supply voltages, it is increasingly distorted because the formula $V_o = A_V V_a$ becomes increasingly inaccurate. So, for linear performance (which is essential for good reproduction of the input signal) amplifiers must be *backed-off* to less than maximum amplification.

Comment: Oddly, some people (such as disc jockeys) set the volume on their hi-fi systems too high and listen to these nonlinear distortions for hours on end. Looking on the bright side, they are benefitting the biomedical engineers who help to design bionic ears.

3.6 Offset Currents and Offset Voltages

Finite gain A_V and input impedance Z_{in} imply that the *offset currents* I_+ and I_- and the *offset voltage* V_a are nonzero. Let's investigate the consequences of each of these separately, for the inverting amplifier circuit.

3.6.1 Effect of the Offset Current

Supposing $V_a = 0$ but $I_- \neq 0$, the currents I_i and I_f through the resistors are related according to Kirchoff's Current Law by:

$$I_i = I_f + I_-$$

Therefore, by Ohm's law,

$$\frac{V_i - 0}{R_i} = \frac{0 - V_o}{R_f} + I_-$$

from which we deduce that

$$V_o = -\frac{R_f}{R_i} V_i + I_- R_f$$

Thus there is a *zero offset* in the output voltage, so the voltage characteristic (curve of V_o versus V_i) is a straight line shifted vertically from the origin by the amount $\Delta V_o = I_- R_f$ which depends on the offset current at the op-amp's negative input.

3.6.2 Effect of the offset voltage

If the offset currents $I_- = I_+ = 0$ but the offset voltage V_a is nonzero, then in the voltage divider we used to calculate the gain, the intermediate value which was zero is replaced by V_a and the current equation becomes:

$$\frac{V_i - V_a}{R_i} = \frac{V_a - V_o}{R_f}$$

Rearranging terms, we obtain

$$\frac{V_o}{R_f} = V_a(\frac{1}{R_i} + \frac{1}{R_f}) - \frac{V_i}{R_i}$$

Multiplying through by R_f and defining ΔV_o as the amount by which V_o differs from its ideal value $-\frac{R_f}{R_i}V_i$ yields:

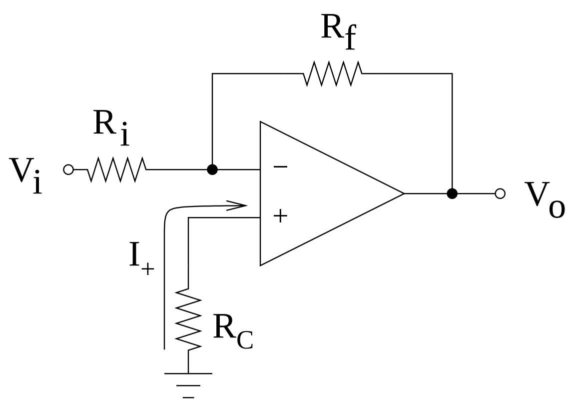
$$\Delta V_o = V_a(1 + \frac{R_f}{R_i})$$

3.7 Compensating for Offsets

A solution to the problem of an offset current or voltage is to use a *pre-offset* to cancel the error ΔV that it caused. Interestingly, a bias voltage is used to compensate for an offset current, and a bias current to compensate for an offset voltage.

We will now consider the specifics of offset compensation for the inverting amplifier circuit.

3.7.1 Bias voltage compensates for offset current



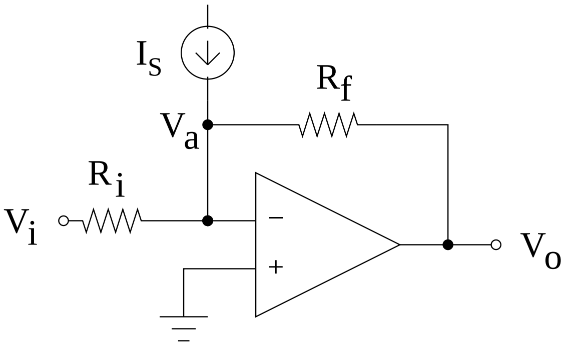
The compensation resistor R_C introduces a *bias* voltage I_+R_C which acts as an offset voltage $V_a = -I_+R_C$ so it will produce a zero offset ΔV_o cancelling that due to offset current I_-provided that:

$$I_+R_C(1 + \frac{R_f}{R_i}) = I_-R_f$$

If we use a variable resistor for R_C we can adjust it until the output voltage's zero offset is nil (or as near as we can achieve); this will occur when:

$$R_C = \frac{I_-}{I_+}(R_i||R_f)$$

3.7.2 Bias current compensates for offset voltage



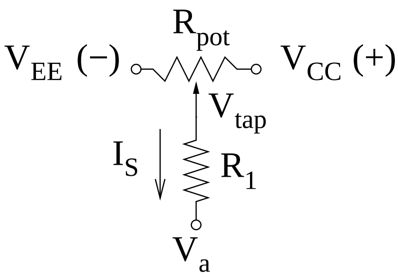
By Kirchoff's Current Law and Ohm's Law, the bias current I_S contributes a term $-I_S R_f$ to the output voltage. The effect of the offset voltage V_a will be cancelled when

$$I_S R_f = V_a(1 + \frac{R_f}{R_i})$$

thus we should choose

$$I_S = \frac{V_a}{R_i || R_f}$$

But how do we adjust the current? One way is to use a potentiometer circuit as the current source:



The voltage V_{tap} tapped by the potentiometer can be varied anywhere in the range from the lower supply V_{EE} to the upper supply V_{CC} . The value of the bias current I_S equals

$$I_S = \frac{V_{tap} - V_a}{R_1}$$

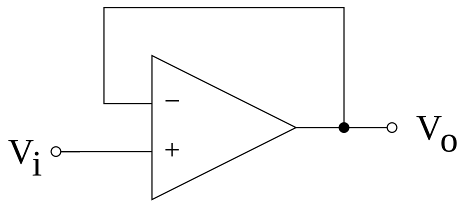
Example: if the power supply rails are$\pm 10V$ and $V_a \approx 0$ and R_1 is 100 kΩ then the magnitude of I_S can be varied up to 0.1A which seems ample. (The diagram in *Carr* has $R_{pot} = 20$ kΩ and R_1 somewhere between 10 kΩ and 100 kΩ, but the particulars are sure to differ between circuits.)

Equating the two formulas for I_S we find that the required tap voltage V_{tap} is proportional to the input offset voltage $V_a = V_-$ that it compensates for:

$$V_{tap} = V_a(1 + \frac{R_1}{R_i || R_f})$$

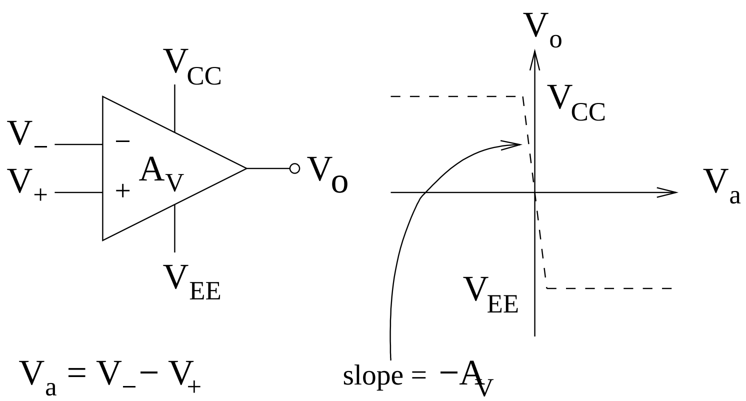
3.8 Basic Applications of the Op-Amp

3.8.1 Voltage Follower



Also known as the Unity Gain Buffer, this is a special case ($R_f = 0$ and $R_i = \infty$ therefore gain $= 1$) of the non-inverting amplifier. Notice that $V_o = V_i$ even though there is no current connecting them. This is a very useful property as it permits *non-ideal voltage sources* (with large resistances) to drive *non-ideal loads* (small resistances) without being *loaded down* (reduced in voltage).

3.8.2 Comparator

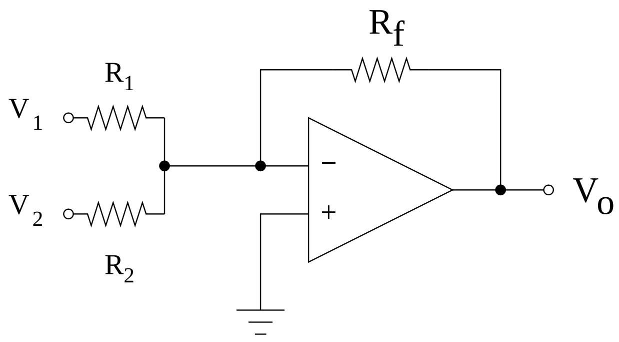


When used directly with inputs that differ significantly so that linearity is completely broken, an op-amp acts as a **comparator**, which is a circuit with analog inputs and an output that is practically digital as it spends most of its time on one or the other of the power supply rails depending on which input is higher:

$$V_+ > V_- \quad \Leftrightarrow \quad V_o = V_{CC}$$
$$V_+ < V_- \quad \Leftrightarrow \quad V_o = V_{EE}$$

3.8.3 Summer

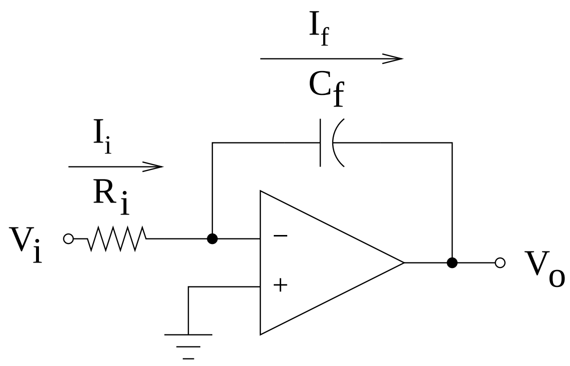
On the other hand, linear operation of the op-amp allows many useful *analog computer* circuits, such as the summer (summation circuit, not the season). This design exploits Ohm's Law and Kirchoff's Current Law.



Note that this summer is essentially an inverting amplifier with two input signals joining at the negative input terminal of the op-amp.

3.8.4 Integrator

The characteristic equations of basic circuit elements allow a variety of analog computation circuits. For example, since current is the time derivative of charge, we can use a resistor ($V = IR$) and a capacitor ($Q = CV$) to integrate current and thereby voltage.

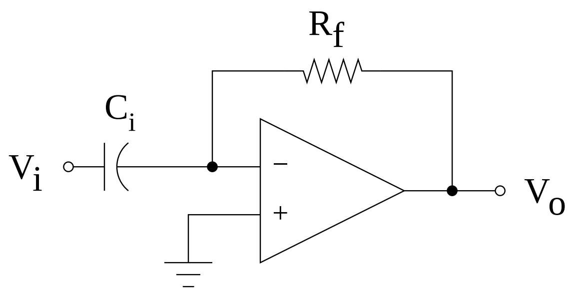


Here, $V_i = I_i R$ and $Q_C = CV_C$ while $I_i = I_f = \int Q_C dt$ so

$$V_o = 0 - V_C = -\frac{Q_C}{C} = -\frac{1}{C}\int I_i dt = -\frac{1}{RC}\int V_i dt$$

3.8.5 Differentiator

By reversing the positions of capacitor and resistor, the integrator is converted into a differentiator:



3.8.6 Logarithmic Amplifier

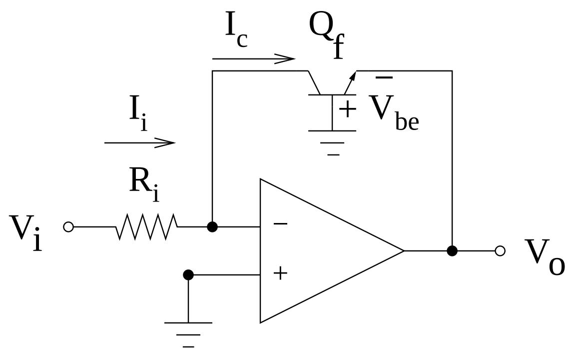
Bipolar Junction Transistors (BJTs) have a logarithmic relationship between collector current I_c and base-emitter voltage V_{be}:

$$V_{be} = V_T \ln(\frac{I_c}{I_s})$$

where I_s = its reverse saturation current. Recall that:

$$V_T = \frac{kT}{q_e}$$

where k = Boltzmann's constant, q_e = the magnitude of the electron charge, T = the absolute temperature of the transistor. We can use this property of BJTs to construct a simple analog circuit that computes logarithms. This is known as a logarithmic amplifier.

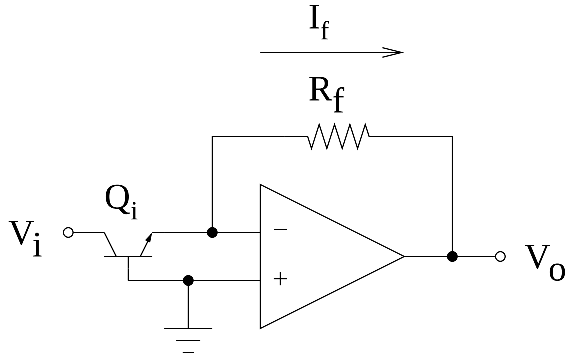


In the circuit above, $V_o = -V_{be}$ and $V_i = I_c R_i$ so (if the offset current is zero) the input-output relation is:

$$V_o = -V_T \ln(\frac{V_i}{I_s R_i})$$

3.8.7 Antilog Amplifier

By exchanging the transistor and the resistor, we obtain a simple antilog amplifier, which calculates the exponential function.



In the forward active region of the transistor, the antilog amplifier's input-output equation is:

$$V_o = I_s R_i \exp(\frac{V_i}{V_T})$$